

June 2005

Final Version

**6679 Mechanics M3
Mark Scheme**

The following abbreviations are used in this scheme.

- M A method mark. These are awarded for 'knowing a method and attempting to apply it'.
A An accuracy mark. Can only be awarded if the relevant method mark(s) have been earned.
B These marks are independent of method marks.
cso correct solution only. There must be no errors in this part of the question to obtain this mark.
cao correct answer only.
ft follow through. The scheme or marking guidance will specify what is to be followed through.
oe or equivalent.
awrt answers which round to

[The second mark is dependent on gaining the first mark.

N2L Newton's second law

LHS Left hand side of an equation

LM Linear momentum

RHS Right hand side of an equation

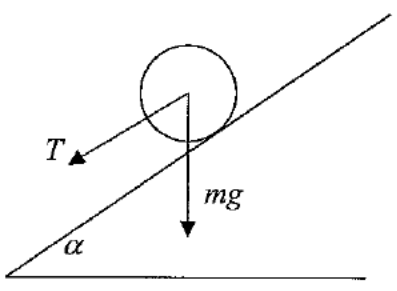
HL Hooke's Law.

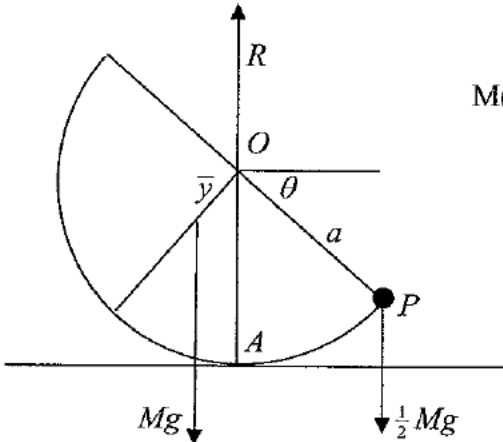
EPE Elastic potential energy

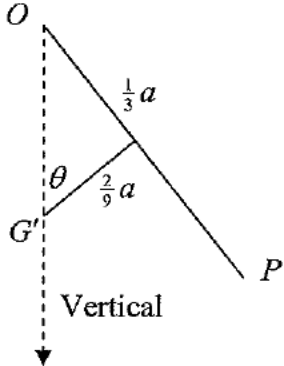
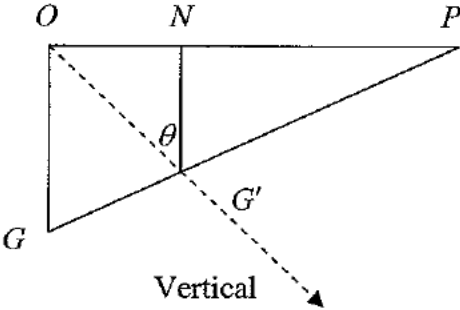
→, ↓ etc. Resolving in the appropriate direction

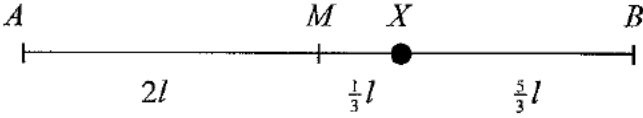
M(*A*) Taking moments about *A*.


* The answer is printed on the paper.

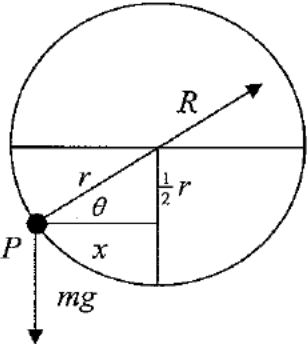
Question Number	Scheme	Marks
1.	 <p data-bbox="782 448 1308 537">HL $T = \frac{20 \times 0.4}{2} (=4)$ accept -4</p> <p data-bbox="782 560 1308 627">[$mg \sin \alpha + T = ma$</p> <p data-bbox="845 649 1308 716">$0.8g \times 0.6 + 4 = 0.8a$</p> <p data-bbox="813 739 1308 806">$a = 10.88 \approx 10.9 \text{ (ms}^{-2}\text{)}$ accept 11</p>	<p data-bbox="1340 470 1436 515">M1 A1</p> <p data-bbox="1340 582 1436 627">M1 A1</p> <p data-bbox="1340 649 1388 694">M1</p> <p data-bbox="1340 761 1388 806">A1</p> <p data-bbox="1468 806 1516 851">[6]</p>

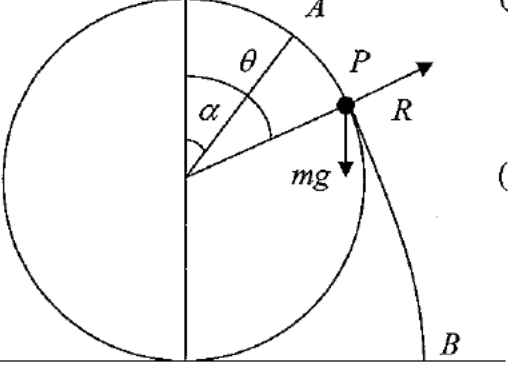
Question Number	Scheme	Marks															
2.	<p>(a)</p> <table border="0" style="width: 100%;"> <tr> <td style="width: 20%;"></td> <td style="width: 20%;">Bowl</td> <td style="width: 20%;">Lid</td> <td style="width: 20%;">C</td> <td style="width: 20%;"></td> </tr> <tr> <td>Mass ratio</td> <td>2</td> <td>1</td> <td>3</td> <td>anything in ratio 2 : 1 : 3</td> </tr> <tr> <td>\bar{y}</td> <td>$\frac{1}{2}a$</td> <td>0</td> <td>\bar{y}</td> <td></td> </tr> </table> <p>$M(O)$ $2 \times \frac{1}{2}a = 3\bar{y}$ $\bar{y} = \frac{1}{3}a$ *</p> <p>(b)</p> <div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p>$M(A)$ $Mg \times \frac{1}{3}a \sin \theta = \frac{1}{2}Mg \times a \cos \theta$ $\tan \theta = \frac{3}{2}$</p> <p>$\theta \approx 56^\circ$</p> </div> </div>		Bowl	Lid	C		Mass ratio	2	1	3	anything in ratio 2 : 1 : 3	\bar{y}	$\frac{1}{2}a$	0	\bar{y}		<p>B1 B1 M1 A1 (4)</p> <p>M1 A1=A1 M1 A1 (5) [9]</p> <p>cs0 cao</p> <p><i>Methods involving the location of the combined centre of mass of C and P are considered on the next page.</i></p>
	Bowl	Lid	C														
Mass ratio	2	1	3	anything in ratio 2 : 1 : 3													
\bar{y}	$\frac{1}{2}a$	0	\bar{y}														

Question Number	Scheme	Marks																
2.	<p>(b) <i>Methods involving the location of the combined centre of mass of C and P.</i></p> <p>G is the centre of mass of C; G' is the combined centre of mass of C and P.</p> <p><i>First Alternative</i></p> <table border="1" data-bbox="236 510 906 674"> <thead> <tr> <th></th> <th>C</th> <th>P</th> <th>C and P</th> </tr> </thead> <tbody> <tr> <td>Mass ratios</td> <td>2</td> <td>1</td> <td>3</td> </tr> <tr> <td>\bar{y}</td> <td>$\frac{1}{3}a$</td> <td>0</td> <td>\bar{y}</td> </tr> <tr> <td>\bar{x}</td> <td>0</td> <td>a</td> <td>\bar{x}</td> </tr> </tbody> </table> <p>Finding both coordinates of G'</p> $\frac{2}{3}a = 3\bar{y} \Rightarrow \bar{y} = \frac{2}{9}a$ $a = 3\bar{x} \Rightarrow \bar{x} = \frac{1}{3}a$  <p style="margin-left: 200px;">Vertical</p> $\tan \theta = \frac{\frac{1}{3}a}{\frac{2}{9}a} = \frac{3}{2}$ $\theta \approx 56^\circ$ <p><i>Second Alternative</i></p>  <p style="margin-left: 200px;">Vertical</p> $GG' : G'P = \frac{1}{2}M : M = 1 : 2$ $OG = \frac{1}{3}a, \quad OP = a$ <p>By similar triangles</p> $ON = \frac{1}{3}OP = \frac{1}{3}a$ $NG' = \frac{2}{3}OG = \frac{2}{9}a$ $\tan \theta = \frac{ON}{NG'} = \frac{\frac{1}{3}a}{\frac{2}{9}a} = \frac{3}{2}$ $\theta \approx 56^\circ$		C	P	C and P	Mass ratios	2	1	3	\bar{y}	$\frac{1}{3}a$	0	\bar{y}	\bar{x}	0	a	\bar{x}	<p>M1 A1 A1</p> <p>M1</p> <p>cao A1</p> <p>(5)</p> <p>M1 A1 A1</p> <p>M1</p> <p>cao A1</p> <p>(5)</p>
	C	P	C and P															
Mass ratios	2	1	3															
\bar{y}	$\frac{1}{3}a$	0	\bar{y}															
\bar{x}	0	a	\bar{x}															

Question Number	Scheme	Marks
3.	<p>(a)</p>  <p>Elastic energy when P is at X: $E = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l} \left(= \frac{40mgl}{9} \right)$</p> $\frac{1}{2}mV^2 + 2 \times \frac{4mgl^2}{2l} = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l}$ $\frac{1}{2}V^2 + 4gl = \frac{8}{9}gl + \frac{32}{9}gl$ $V^2 = \frac{8gl}{9} \quad \text{solving for } V^2$ $V = \left(\frac{8gl}{9}\right)^{\frac{1}{2}} \quad \text{or exact equivalents}$ <p>(b) The maximum speed occurs when $a = 0$ At M the particle is in equilibrium (the sum of the forces is zero) $\Rightarrow a = 0$</p> <p><i>The alternative method using Newton's Second Law is considered on the next page.</i></p>	<p>M1 A1</p> <p>M1A1=A1ft</p> <p>M1</p> <p>A1 (7)</p> <p>B1</p> <p>B1 (2)</p> <p>[9]</p>

Question Number	Scheme	Marks
3.	<p>Alternative using Newton's second law.</p> <p>(a)</p>  <p>HL $T_1 = \frac{4mg(l+x)}{l}, T_2 = \frac{4mg(l-x)}{l}$</p> <p>N2L $m\ddot{x} = T_2 - T_1 = -\frac{8mg}{l}x$</p> <p>This is SHM, centre M</p> <p>$a = \frac{l}{3}, \omega^2 = \frac{8g}{l}$</p> <p>$v^2 = \omega^2(a^2 - x^2) \Rightarrow v^2 = \frac{8g}{l} \left(\frac{l^2}{9} - x^2 \right)$ Depends on showing SHM</p> <p>At M, $x = 0, V^2 = \frac{8gl}{9}, V = \left(\frac{8gl}{9} \right)^{\frac{1}{2}}$ or exact equivalents</p> <p>(b) The particle is performing SHM about the mid-point of AB. The maximum speed occurs at the centre of the oscillation (when $x = 0$)</p>	<p>M1 A1</p> <p>A1, A1ft</p> <p>M1</p> <p>M1, A1 (7)</p> <p>B1</p> <p>B1 (2)</p> <p>[9]</p>

Question Number	Scheme	Marks
4.	<div style="display: flex; align-items: center;">  <div style="margin-left: 20px;"> <p>(a) $\sin \theta = \frac{\frac{1}{2}r}{r} = \frac{1}{2} \quad (\Rightarrow \theta = 30^\circ)$</p> <p>$\uparrow \quad R \sin \theta = mg$ $R = 2mg$</p> <p>(b) $\rightarrow \quad R \cos \theta = m x \omega^2$ $= m (r \cos \theta) \omega^2$</p> <p>$\omega = \left(\frac{2g}{r} \right)^{\frac{1}{2}}$</p> <p>$T = \frac{2\pi}{\omega} = 2\pi \left(\frac{r}{2g} \right)^{\frac{1}{2}}$ or exact equivalent</p> </div> </div> <p>Note: $x = \frac{\sqrt{3}}{2} r$</p>	<p>B1</p> <p>M1 A1 A1 (4)</p> <p>M1 A1 A1</p> <p>A1</p> <p>M1 A1 (6)</p> <p>[10]</p>

Question Number	Scheme	Marks
5.		(a) $\frac{1}{2}mv^2 = mg(a \cos \alpha - a \cos \theta)$ $v^2 = 2ga(\cos \alpha - \cos \theta)$ * cso M1 A1 A1 A1 (4)
	(b) $mg \cos \theta - R = \frac{mv^2}{a}$ ($R = 0$) $g \cos \theta = 2g\left(\frac{3}{4} - \cos \theta\right)$ $\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ (accept 60°) M1 A1=A1 M1 A1 (5)	
(c) From A to B $\frac{1}{2}mw^2 = mg(a + a \cos \alpha)$ $w^2 = 2ga\left(1 + \frac{3}{4}\right) \Rightarrow w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}$ M1 A1 A1 A1 (4)	[13]	

Alternative solutions to 5(c) are considered on the next page.

Question Number	Scheme	Marks
5.	<p><i>Alternatives to 5(c)</i></p> <p><i>From P to C</i></p> $v_p^2 = 2ga \left(\frac{3}{4} - \frac{1}{2} \right) = \frac{ga}{2}$ $\frac{1}{2}mw^2 - \frac{1}{2}m \left(\frac{ga}{2} \right) = mg(a + a \cos \theta)$ $w^2 - \frac{ga}{2} = 2mga \left(1 + \frac{1}{2} \right) \Rightarrow w = \left(\frac{7ga}{2} \right)^{\frac{1}{2}}$ <p><i>Alternatives using projectile motion from P</i></p> $v_p = \left(\frac{ga}{2} \right)^{\frac{1}{2}}, \text{ as above}$ $\downarrow u_y = \left(\frac{ga}{2} \right)^{\frac{1}{2}} \sin 60^\circ = \left(\frac{3ga}{8} \right)^{\frac{1}{2}}$ $\downarrow v_y^2 = u_y^2 + 2g \times \frac{3a}{2} = \frac{27ga}{8}$ $\rightarrow u_x = \left(\frac{ga}{2} \right)^{\frac{1}{2}} \cos 60^\circ = \left(\frac{ga}{8} \right)^{\frac{1}{2}}$ $w^2 = u_x^2 + v_y^2 = \frac{ga}{8} + \frac{27ga}{8} = \frac{7ga}{2} \Rightarrow w = \left(\frac{7ga}{2} \right)^{\frac{1}{2}}$ <p><i>There are also longer projectile methods using time of flight</i></p> <p>In outline, solving $\frac{3a}{2} = \left(\frac{3ga}{8} \right)^{\frac{1}{2}} t + \frac{1}{2}gt^2$ gives $t = \left(\frac{3a}{2g} \right)^{\frac{1}{2}}$,</p> <p>then, using $v = u + at$ gives $v_y = \left(\frac{3ga}{8} \right)^{\frac{1}{2}} + g \left(\frac{3a}{2g} \right)^{\frac{1}{2}} = \left(\frac{27ga}{8} \right)^{\frac{1}{2}}$, then as before.</p>	<p>M1 A1 A1</p> <p>A1 (4)</p> <p>M1, A1</p> <p>A1</p> <p>A1 (4)</p> <p>M1 A1</p>

Question Number	Scheme	Marks
6.	<p>(a) $a = 3, T = 12$ (or $\frac{1}{2}T = 6$)</p> $T = \frac{2\pi}{\omega} = 12 \Rightarrow \omega = \frac{\pi}{6} \quad (\square 0.52)$ <p>In the scheme below, when a and/or ω appear in a line, accept the symbols or the candidates' values of a and/or ω for the marks in that line.</p> <p>(Taking $x = a$ when $t = 0$) $x = a \cos \omega t$</p> $\dot{x} = -a\omega \sin \omega t$ <p>When $t = 5$ $\dot{x} = -3 \times \frac{\pi}{6} \sin \frac{5\pi}{6}$</p> $ \dot{x} = \frac{\pi}{4} \quad (\text{mh}^{-1})$	<p>B1, B1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>awrt 0.79 A1 (9)</p>
	<p>(b) Depth of 5.5 m $\Rightarrow x = -1.5$</p> $-1.5 = a \cos \omega t$ $\cos \omega t = -\frac{1}{2}$ $\frac{\pi}{6}t = \frac{2\pi}{3}, \left(\frac{4\pi}{3}\right)$ $t = 4, 8$ <p>Required time is $t_2 - t_1 = 8 - 4 = 4$ (h)</p> <p>In 6(b), the following should be accepted</p> $1.5 = a \cos \omega t$ $\cos \omega t = \frac{1}{2}$ $\frac{\pi}{6}t = \frac{\pi}{3}$ $t = 2$ <p>Required time is $2t = 4$ (h)</p> <p><i>Further alternatives are given over the page.</i></p>	<p>M1</p> <p>A1ft</p> <p>M1</p> <p>A1</p> <p>A1 (5)</p> <p>[14]</p> <p>M1</p> <p>A1ft</p> <p>M1</p> <p>A1</p> <p>A1 (5)</p>

Question Number	Scheme	Marks
<p>6.</p>	<p><i>Alternative to 6(a)</i> The last 5 marks of 6(a) can be gained as follows. The first 4 marks are as above.</p> <p>When $t = 5$</p> $x = 3 \cos \frac{5\pi}{6} = -\frac{3\sqrt{3}}{2} \quad (\square -2.60)$ $v^2 = \omega^2 (a^2 - x^2)$ $= \frac{\pi^2}{6^2} \left(9 - \frac{9 \times 3}{4} \right) \quad \left(= \frac{\pi^2}{16} \right)$ $ v = \frac{\pi}{4} \quad (\text{m h}^{-1})$ <p><i>Alternatives measuring x from the centre of oscillation</i></p> <p>(a) (Using 1400 as $t = 0$) The first 4 marks are as above</p> $x = a \sin \omega t$ $\dot{x} = a\omega \cos \omega t$ <p>When $t = 2$</p> $\dot{x} = 3 \times \frac{\pi}{6} \cos \frac{2\pi}{6} \quad t=2 \text{ oe is essential for this M}$ $= \frac{\pi}{4} \quad (\text{m h}^{-1})$ <p>(b)</p> $1.5 = 3 \sin \omega t$ $\sin \omega t = \frac{1}{2}$ $\frac{\pi}{6} t = \frac{\pi}{6}, \quad \left(\frac{5\pi}{6} \right)$ $t = 1, 5$ <p>Required time is $t_2 - t_1 = 5 - 1 = 4 \quad (\text{h})$</p>	<p>M1</p> <p>M1</p> <p>M1 A1</p> <p>awrt 0.79 A1</p> <p>B1 B1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (9)</p> <p>M1</p> <p>A1ft</p> <p>M1</p> <p>A1</p> <p>A1 (5)</p> <p>[14]</p>

Question Number	Scheme	Marks
7.	(a) $\frac{1}{3}\ddot{x} = -\frac{k}{(x+1)^2}$ $\frac{1}{3}v \frac{dv}{dx} = -\frac{k}{(x+1)^2}$ $\int v dv = \int -\frac{3k}{(x+1)^2} dx \quad \text{Separating variables \&}$ $\frac{1}{2}v^2 = \frac{3k}{x+1} (+C) \quad \text{attempting integration of both sides}$ $v^2 = \frac{6k}{x+1} + A$	M1 M1 M1 A1=A1
	Using boundary values to obtain two simultaneous equations. $(1, 4) \quad 16 = 3k + A$ $(8, \sqrt{2}) \quad 2 = \frac{2k}{3} + A$ $14 = \frac{7}{3}k \Rightarrow k = 6$	M1 A1 A1 M1 A1 (10)
	(b) $A = -2$ $v^2 = \frac{36}{x+1} - 2 = 0$ $x = 17 \text{ (m)}$	B1 M1 MI A1 (4) [14]